THE SCIENTIFIC WORKS OF PHILIPPE FLAJOLET

Hsien-Kuei Hwang
Institute of Statistical Science, Academia Sinica

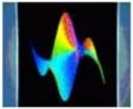
June 15, 2011

An outsider's view

$\downarrow$

A guided tour (by themes)

PHILIPPE FLAJOLET'S WEBPAGE



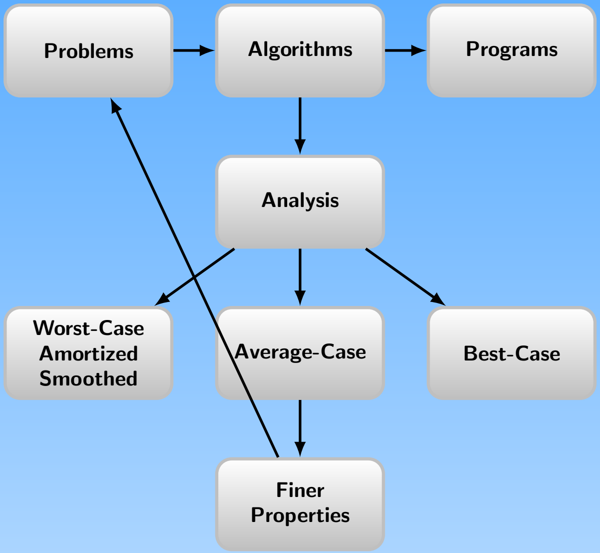
Philippe Flajolet's Home Page

Philippe Flajolet died suddendly on March 22, 2011. This page will be left untouched.

``Does research. Smokes. Battles administration. Smokes. Wishes he could stop battling administration so that he could have more time to do research. Smokes some more. Gives jobs to starving foreigners. Eats occassionally.'' --- Eithne Murray

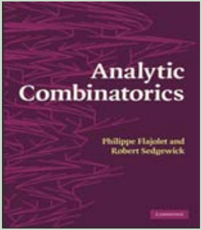
Interests: *Analysis of algorithms*, analytic combinatorics (and *vice versa*), computer algebra, asymptotic analysis, special functions, random structures, natural languages.

ANALYSIS OF ALGORITHMS



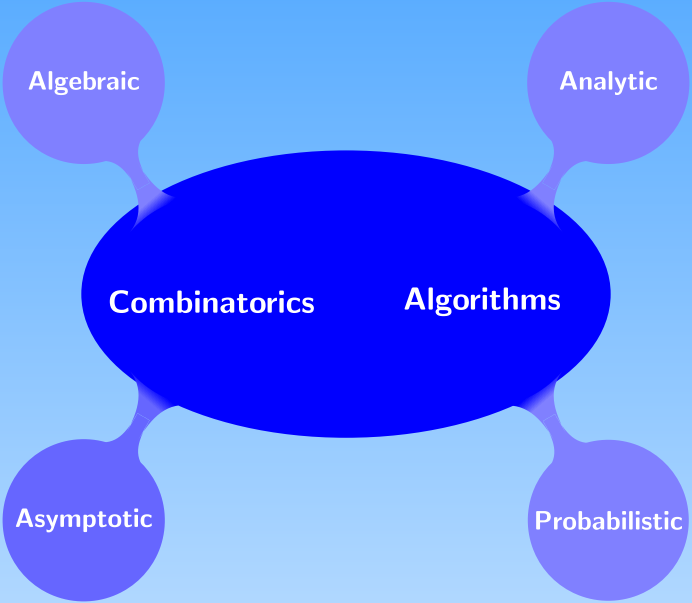
ANALYTIC COMBINATORICS

Flajolet and Sedgewick, Analytic Combinatorics, 2009



Analytic combinatorics aims to enable precise quantitative predictions of the properties of large combinatorial structures. The theory has emerged over recent decades as essential both for the analysis of algorithms and for the study of scientific models in many disciplines, including probability theory, statistical physics, computational biology and information theory. With a careful combination of symbolic enumeration methods and complex analysis, drawing heavily on generating functions, results of sweeping generality emerge that can be applied in particular to fundamental structures such as permutations, sequences, strings, walks, paths, trees, graphs and maps.

ALGORITHMS AND COMBINATORICS



KNUTH, THE ART OF COMPUTER PROGRAMMING I

1.2.10. Analysis of an Algorithm

Let us now apply some of the techniques of the preceding sections to the study of a typical algorithm.

Algorithm M (*Find the maximum*). Given n elements $X[1], X[2], \ldots, X[n]$, we will find m and j such that $m = X[j] = \max_{1 \le i \le n} X[i]$, where j is the largest index that satisfies this relation.

M1. [Initialize.] Set $j \leftarrow n$, $k \leftarrow n-1$, $m \leftarrow X[n]$. (During this algorithm we will have $m = X[j] = \max_{k < i \le n} X[i]$.) 1

M2. [All tested?] If $k = 0$, the algorithm terminates. n

M3. [Compare.] If $X[k] \le m$, go to M5. n-1

M4. [Change m.] Set $j \leftarrow k$, $m \leftarrow X[k]$. (This value of m is a new current maximum.) Y

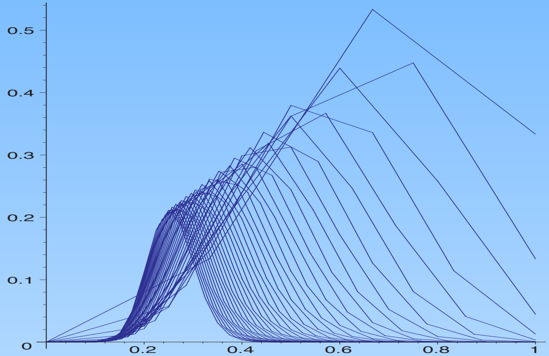
M5. [Decrease k.] Decrease k by one and return to M2. ▮ n-1

Analysis of Y

$$Y = \{0, 1, \ldots, n-1\}$$

What's the mean value, assuming random permutations?

Mean $\sim \log n$, **Variance** $\sim \log n$



MAXIMA-FINDING ALGORITHM

$Y_n :=$ number of records (left-to-right max) in a random permutation of n elements

$$\mathbb{E}(z^{Y_n}) = \frac{z(z+1)\cdots(z+n-1)}{n!} \quad \textbf{(Stirling \#s 1st kind)}$$

$$\mathbb{E}(Y_n) = \sum_{1 \le j \le n} \frac{1}{j} = \log n + \gamma + \cdots$$

$$\mathbb{V}(Y_n) = \sum_{1 \le j \le n} \left(\frac{1}{j} - \frac{1}{j^2}\right) = \log n + \gamma - \frac{\pi^2}{6} + \cdots$$

$$\frac{Y_n - \log n}{\sqrt{\log n}} \to \mathcal{N}(0,1)$$

$$\mathbb{P}\left(Y_n = \left\lfloor \log n + x\sqrt{\log n} \right\rfloor\right) = \frac{e^{-x^2/2}}{\sqrt{2\pi \log n}}\left(1 + O\left(\frac{1}{\sqrt{\log n}}\right)\right)$$

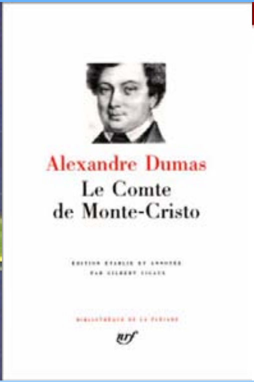
MARCH 5, 2010 (NORTHEAST COAST, TAIWAN)

MARCH 5, 2010 (NORTHEAST COAST, TAIWAN)

MARCH 5, 2010 (NORTHEAST COAST, TAIWAN)

ALEXANDRE DUMAS & PHILIPPE FLAJOLET

ALEXANDRE DUMAS (1802–1870): FAMOUS NOVELS



ALEXANDRE DUMAS & PHILIPPE FLAJOLET

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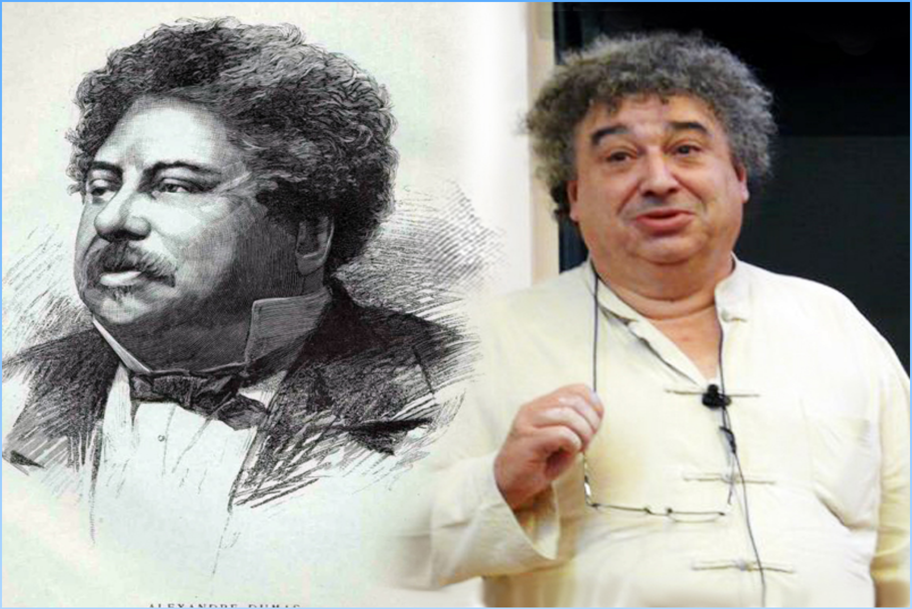

ALEXANDRE DUMAS & PHILIPPE FLAJOLET

ALEXANDRE DUMAS & PHILIPPE FLAJOLET

ALEXANDRE DUMAS & PHILIPPE FLAJOLET

EVENTS IN THE LIFE OF PHILIPPE

1948 **Born in Lyon**

1966 **Baccalauréat**

1968

Ecole polytechnique

1970

1971 **DEA; Research Assistant at INRIA**

1973 **Thèse 3e cycle, Université Paris 7**

Docteur ès Science, Université Paris 11

1979 **Research Director**

1981 **Head of Projet ALGO**

1986 **Prix Scientifique (Union des Assurances de Paris)**

Prix Michel Montpetit

1994 **corresponding member (l'Académie des Sciences)**

Doctorate Honoris Causa (U. Libre Bruxelles)

1996 **Analysis of Algorithms book**

Fellow, Academia Europaea

2003 **Full member l'Académie des Sciences**

2004 **Médaille d'argent CNRS**

2009 **Analytic Combinatorics book published**

2011 **Rest in peace**

AMS MATH REVIEWS

AMERICAN MATHEMATICAL SOCIETY
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Academia Sinica

Flajolet, Philippe

MR Author ID: **67375**
Earliest Indexed Publication: 1973
Total Publications: **149**
Total Author/Related Publications: **156**
Total Citations: **1546**

⊞ Also published as: Flajolet, P....

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Top 50 Co-authors (by number of collaborations)

Banderier, Cyril · Clément, Julien · Coffman, Edward G., Jr. · Daudé, Hervé · Denise, Alain · Devroye, Luc P. · Duchon, Philippe · Dumas, Philippe · Fayolle, Guy · Françon, Jean · Fusy, Éric · Gabarró, Joaquim · Gardy, Danièle · Gerhold, Stefan · Golin, Mordecai J. · Gonnet, Gaston H. · Gourdon, Xavier · Grabner, Peter J. · Hofri, Micha · Hurtado, Ferran · Jacquet, Philippe A. · Kirschenhofer, Peter · Laffogue, T. · Louchard, Guy · Nebel, Markus E. · Nicodème, Pierre · Nikoletseas, Sotiris E. · Noy, Marc · Odlyzko, Andrew M. · Panario, Daniel · Prodinger, Helmut · Puech, Claude · Raoult, Jean-Claude · Régnier, Mireille · Richmond, L. Bruce · Robson, John Michael · Saheb-Djahromi, Nasser · Salvy, Bruno · Schaeffer, Gilles · Sedgewick, Robert · Soria, Michèle · Steyaert, Jean-Marc · Szpankowski, Wojciech · Thimonier, Loÿs · Tichy, Robert Franz · Vallée, Brigitte · Van Cutsem, Bernard · van Fossen Conrad, Eric · Vuillemin, Jean E. · Zimmermann, Paul

See All

Publications (by number in area)

Combinatorics · Computer science · Convex and discrete geometry · Difference and functional equations · History and biography · Information and communication, circuits · Integral transforms, operational calculus · Logic and foundations · Number theory · Numerical analysis · Probability theory and stochastic processes · Special functions · Statistical mechanics, structure of matter

Publications (by number of citations)

Combinatorics · Computer science · Convex and discrete geometry · Difference and functional equations · Information and communication, circuits · Integral transforms, operational calculus · Logic and foundations · Number theory · Numerical analysis · Probability theory and stochastic processes · Special functions

MOST CITED PAPERS (AMS MATHSCINET)

AMERICAN MATHEMATICAL SOCIETY MathSciNet® Mathematical Reviews on the Web

Author Citations for Philippe Flajolet
Philippe Flajolet is cited 1546 times by 892 authors
in the MR Citation Database

Citations	Publication
165	MR1039294 (90m:05012) Flajolet, Philippe; Odlyzko, Andrew Singularity analysis of generating functions. SIAM J. Discrete Math. 3 (1990), no. 2, 216-240. (Reviewer: E. Rodney Canfield), 05A15 (30E20 40E05 41A60)
105	MR2483235 (2010h:05005) Flajolet, Philippe; Sedgewick, Robert Analytic combinatorics. Cambridge University Press, Cambridge, 2009. xiv+810 pp. ISBN: 978-0-521-89806-5, 05-02 (05A15 05A16 60C05 60E10 82-01)
77	MR1337752 (96h:68093) Flajolet, Philippe; Gourdon, Xavier; Dumas, Philippe Mellin transforms and asymptotics: harmonic sums. Special volume on mathematical analysis of algorithms. Theoret. Comput. Sci. 144 (1995), no. 1-2, 3-58. (Reviewer: Peter Kirschenhofer), 68Q25 (44A15 68P05)
64	MR0592851 (82f:05002a) Flajolet, P. Combinatorial aspects of continued fractions. Discrete Math. 32 (1980), no. 2, 125-161. (Reviewer: L. Carlitz), 05A10 (05A15 30B70)
50	MR1884885 (2003c:05008) Banderier, Cyril; Bousquet-Mélou, Mireille; Denise, Alain; Flajolet, Philippe; Gardy, Daniéle; Gouyou-Beauchamps, Dominique Generating functions for generating trees. Formal power series and algebraic combinatorics (Barcelona, 1999). Discrete Math. 246 (2002), no. 1-3, 29-55. (Reviewer: Mark Curtis Wilson), 05A15 (05C05)
39	MR1691870 (2000c:05012) Flajolet, Philippe; Noy, Marc Analytic combinatorics of non-crossing configurations. Discrete Math. 204 (1999), no. 1-3, 203-229. (Reviewer: Edward A. Bender), 05A16
38	MR1337755 (96i:39003) Flajolet, Philippe; Sedgewick, Robert Mellin transforms and asymptotics: finite differences and Rice's integrals. Special volume on mathematical analysis of algorithms. Theoret. Comput. Sci. 144 (1995), no. 1-2, 101-124. (Reviewer: M. Mendés France), 39A10 (44A15 68P05 68Q25)
37	MR1290534 (96f:05172) Flajolet, Philippe; Zimmerman, Paul; Van Cutsem, Bernard A calculus for the random generation of labelled combinatorial structures. Theoret. Comput. Sci. 132 (1994), no. 1-2, 1-35. (Reviewer: Norbert Blum), 05C80 (68R05)
33	MR0680517 (84a:68056) Flajolet, Philippe; Odlyzko, Andrew The average height of binary trees and other simple trees. J. Comput. System Sci. 25 (1982), no. 2, 171-213, 68E10 (05C05)
33	MR1251994 (94j:68233) Bergeron, François; Flajolet, Philippe; Salvy, Bruno Varieties of increasing trees. CAAP '92 (Rennes, 1992), 24-48, Lecture Notes in Comput. Sci., 581, Springer, Berlin, 1992, 68R05 (05C05 05C85 68R10)

MOST CITED PAPERS (WEB OF SCIENCE)

1	SINGULARITY ANALYSIS OF GENERATING-FUNCTIONS	SIAM JOURNAL ON DISCRETE MATHEMATICS	1990	327
2	PROBABILISTIC COUNTING ALGORITHMS FOR DATABASE APPLICATIONS	JOURNAL OF COMPUTER AND SYSTEM SCIENCES	1985	205
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4	MELLIN TRANSFORMS AND ASYMPTOTICS - HARMONIC SUMS	THEORETICAL COMPUTER SCIENCE	1995	122
5	THE AVERAGE HEIGHT OF BINARY-TREES AND OTHER SIMPLE TREES	JOURNAL OF COMPUTER AND SYSTEM SCIENCES	1982	105
6	A CALCULUS FOR THE RANDOM GENERATION OF LABELED COMBINATORIAL STRUCTURES	THEORETICAL COMPUTER SCIENCE	1994	83
7	Q-ARY COLLISION RESOLUTION ALGORITHMS IN RANDOM-ACCESS SYSTEMS WITH FREE OR BLOCKED CHANNEL ACCESS	IEEE TRANSACTIONS ON INFORMATION THEORY	1985	71
8	DIGITAL SEARCH-TREES REVISITED	SIAM JOURNAL ON COMPUTING	1986	70
9	BIRTHDAY PARADOX, COUPON COLLECTORS, CACHING ALGORITHMS AND SELF-ORGANIZING SEARCH	DISCRETE APPLIED MATHEMATICS	1992	69
10	RANDOM MAPPING STATISTICS	LECTURE NOTES IN COMPUTER SCIENCE	1990	68
11	ON THE PERFORMANCE EVALUATION OF EXTENDIBLE HASHING AND TRIE SEARCHING	ACTA INFORMATICA	1983	67
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13	PARTIAL MATCH RETRIEVAL OF MULTIDIMENSIONAL DATA	JOURNAL OF THE ACM	1986	63
14	ESTIMATING THE MULTIPLICITIES OF CONFLICTS TO SPEED THEIR RESOLUTION IN MULTIPLE ACCESS CHANNELS	JOURNAL OF THE ACM	1987	60
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16	MELLIN TRANSFORMS AND ASYMPTOTICS - FINITE-DIFFERENCES AND RICES INTEGRALS	THEORETICAL COMPUTER SCIENCE	1995	59
17	GENERATING FUNCTIONS FOR GENERATING TREES	DISCRETE MATHEMATICS	2002	54
18	ANALYTIC COMBINATORICS OF NON-CROSSING CONFIGURATIONS	DISCRETE MATHEMATICS	1999	46
19	ON THE ANALYSIS OF LINEAR PROBING HASHING	ALGORITHMICA	1998	46
20	MELLIN TRANSFORMS AND ASYMPTOTICS - DIGITAL SUMS	THEORETICAL COMPUTER SCIENCE	1994	45

MOST CITED PAPERS (GOOGLE SCHOLAR)

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Citations	Publication	Citations	Publication
727	**[BOOK] Analytic combinatorics** P Flajolet... - 2009 - books.google.com ANALYTIC COMBINATORICS Analytic combinatorics aims to enable precise quantitative predictions of the proper- ties of large combinatorial structures. The theory has emerged over recent decades as essential both for the analysis of algorithms and for the study of scientific models in ... Cited by 727 - Related articles - All 10 versions	256	**Mellin transforms and asymptotics: Harmonic sums** P Flajolet, X Gourdon... - Theoretical computer science, 1995 - Elsevier This survey presents a unified and essentially self-contained approach to the asymptotic analysis of a large class of sums that arise in combinatorial mathematics, discrete probabilistic models, and the average-case analysis of algorithms. It relies on the Mellin transform, a ... Cited by 256 - Related articles - BL Direct - All 28 versions
622	**[BOOK] Singularity analysis of generating functions** P Flajolet... - 1988 - Citeseer SINGULARITY ANALYSIS OF GENERATING FUNCTIONS Philippe Flajolet and Andrew Odlyzko Abstract: This work presents a class of methods by which one can translate, on a term-by-term basis, an asymptotic expansion of a function around a dominant singularity into a ... Cited by 622 - Related articles - All 16 versions	208	**The average height of binary trees and other simple trees** P Flajolet... - Journal of Computer and System Sciences, 1982 - Elsevier The average height of a binary tree with n internal nodes is shown to be asymptotic to $2\pi n$. This represents the average stack height of the simplest recursive tree traversal algorithm. The method used in this estimation is also applicable to the analysis of ... Cited by 208 - Related articles - All 12 versions
598	**Probabilistic counting algorithms for data base applications** P Flajolet... - Journal of Computer and System Sciences, 1985 - Elsevier This paper introduces a class of probabilistic counting algorithms with which one can estimate the number of distinct elements in a large collection of data (typically a large file stored on disk) in a single pass using only a small additional storage (typically less than a hundred ... Cited by 598 - Related articles - All 28 versions	187	**A calculus for the random generation of labelled combinatorial structures** P Flajolet, P Zimmerman... - Theoretical Computer Science, 1994 - Elsevier A systematic approach to the random generation of labelled combinatorial objects is presented. It applies to structures that are decomposable, ie, formally specifiable by grammars involving set, sequence, and cycle constructions. A general strategy is developed for ... Cited by 187 - Related articles - All 3 versions
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298	**Combinatorial aspects of continued fractions** P Flajolet - Discrete Mathematics, 1980 - Elsevier We show that the universal continued fraction of the Stieltjes-Jacobi type is equivalent to the characteristic series of labelled paths in the plane. The equivalence holds in the set of series in non-commutative indeterminates. Using it, we derive direct combinatorial proofs of ... Cited by 298 - Related articles - All 12 versions	171	**Q-ary collision resolution algorithms in random-access systems with free or blocked channel access** ...P Flajolet - Information Theory, IEEE Transactions..., 1985 - ieeexplore... Abstract-The throughput characteristics of contention-based random- access systems @AS's) which use Q-ary tree algorithms (where Q 2 2 is the number of groups into which contending users are split) of the Capetanakis-Tsybakov-Mikhailov-Vvedenskaya type are analyzed ... Cited by 171 - Related articles - All 14 versions

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Author impact analysis - Perform a citation analysis for one or more authors

Author's name: "p flajolet"

Exclude these names:

Year of publication between: 1970 and: 2011

- [x] Biology, Life Sciences, Envir
- [x] Business, Administration, Fir
- [x] Chemistry and Materials Scie
- [x] Engineering, Computer Scier
- [x] Medicine, Pharmacology, Ve
- [x] Physics, Astronomy, Planeta
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Results

Papers:	454	Cites/paper:	22.56	h-index:	52	AWCR:	818.36
Citations:	10241	Cites/author:	5080.39	g-index:	94	AW-index:	28.61
Years:	40	Papers/author:	222.91	hc-index:	24	AWCRpA:	386.94
Cites/year:	256.03	Authors/paper:	2.50	hI-index:	21.98	e-index:	69.19
				hI,norm:	33	hm-index:	34.83

Cites	Per year	Rank	Authors	Title	Year	Publication	Publisher
736	245.33	1	P Flajolet...	Analytic combinatorics	2009		books.google.com
628	26.17	2	P Flajolet...	Singularity analysis of generating functions	1988		Citeseer
600	22.22	3	P Flajolet...	Probabilistic counting algorithms for data base applications	1985	Journal of Computer and Sy...	Elsevier
483	30.19	4	..., P Flajolet	An Introduction to the Analysis of Alforithms	1996		Pearson Education India
299	9.34	5	P Flajolet	Combinatorial aspects of continued fractions	1980	Discrete Mathematics	Elsevier
258	15.18	6	P Flajolet, ...	Meilin transforms and asymptotics: Harmonic sums	1995	Theoretical computer science	Elsevier
208	6.93	7	P Flajolet...	The average height of binary trees and other simple trees	1982	Journal of Computer and Sy...	Elsevier
188	10.44	8	P Flajolet, ...	A calculus for the random generation of labelled combin...	1994	Theoretical Computer Science	Elsevier
171	6.33	9	..., P Flajolet	Q-ary collision resolution algorithms in random-access s...	1985	Information Theory, IEEE Tr...	ieeexplore.ieee.org
171	7.77	10	P Flajolet...	Random mapping statistics	1990	Advances in cryptology—EU...	Springer
162	7.36	11	..., P Flajolet	Average-case analysis of algorithms and data structures	1990		Citeseer
136	4.86	12	P Flajolet...	Digital search trees revisited	1984		hal.inria.fr
136	6.80	14	P Flajolet, ...	Birthday paradox, coupon collectors, caching algorithms...	1992	Discrete Applied Mathematics	Elsevier
133	4.59	13	P Flajolet	On the performance evaluation of extendible hashing a...	1983	Acta informatica	Springer
127	15.88	16	C Banderie...	Generating functions for generating trees	2004	Arxiv preprint math/ ...	arxiv.org
125	13.89	15	..., P Flajolet	Loglog counting of large cardinalities	2003	Algorithms-ESA 2003	Springer
123	7.24	17	P Flajolet...	Mellin transforms and asymptotics: Finite differences an...	1995	Theoretical Computer Science	Elsevier
118	4.72	19	P Flajolet	Analytic models and ambiguity of context-free language...	1987	Theoretical Computer Science	Elsevier
111	5.05	18	P Flajolet, ...	Automatic average-case analysis of algorithms	1990		hal.inria.fr
110	3.79	22	P Flajolet...	Probabilistic counting	1983	Foundations of Computer Sc...	ieeexplore.ieee.org

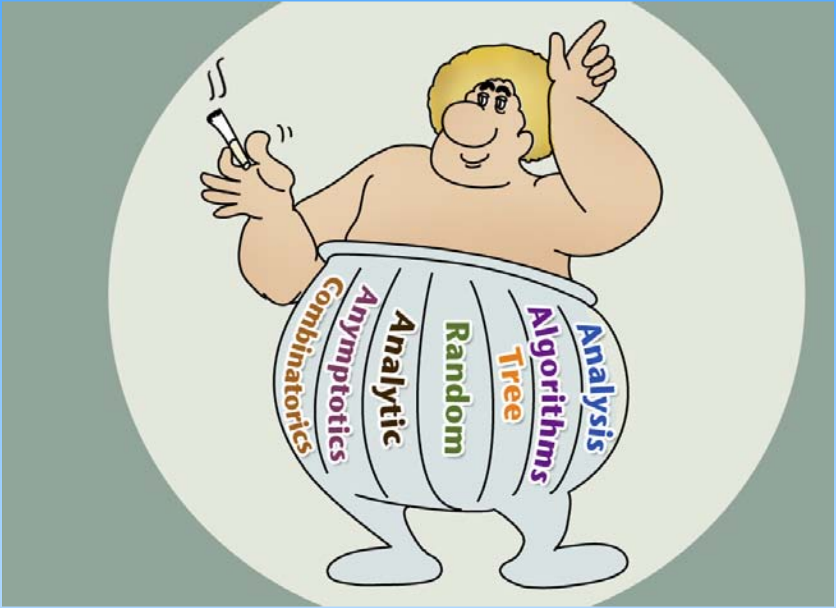
FREQUENT WORDS IN TITLES

Rank	Frequency	%	Rank	Frequency	%	Rank	Frequency	%
1. algorithm	54	25.84%	18. distribution	9	4.31%	35. polynomial	6	2.87%
2. analysis	44	21.05%	19. binary	8	3.83%	36. dynamic	6	2.87%
3. tree	42	20.10%	20. number	8	3.83%	37. mathematics	6	2.87%
4. structure	24	11.48%	21. recursive	8	3.83%	38. limit	6	2.87%
5. random	22	10.53%	22. counting	7	3.35%	39. models	5	2.39%
6. analytic	20	9.57%	23. digital	7	3.35%	40. theory	5	2.39%
7. asymptotics	16	7.66%	24. variations	7	3.35%	41. trie	5	2.39%
8. function	15	7.18%	25. process	7	3.35%	42. language	5	2.39%
9. combinatorial	13	6.22%	26. singularity	7	3.35%	43. sequence	5	2.39%
10. combinatorics	12	5.74%	27. finite	6	2.87%	44. files	5	2.39%
11. average-case	11	5.26%	28. mellin	6	2.87%	45. reduction	5	2.39%
12. data	11	5.26%	29. statistics	6	2.87%	46. probabilistic	5	2.39%
13. complexity	11	5.26%	30. calculus	6	2.87%	47. evaluation	5	2.39%
14. continued	10	4.78%	31. sum	6	2.87%	48. airy	4	1.91%
15. fraction	10	4.78%	32. transform	6	2.87%	49. computer	4	1.91%
16. search	10	4.78%	33. problèmes	6	2.87%	50. fichiers	4	1.91%
17. generating	9	4.31%	34. quadtree	6	2.87%	51. gaussian	4	1.91%

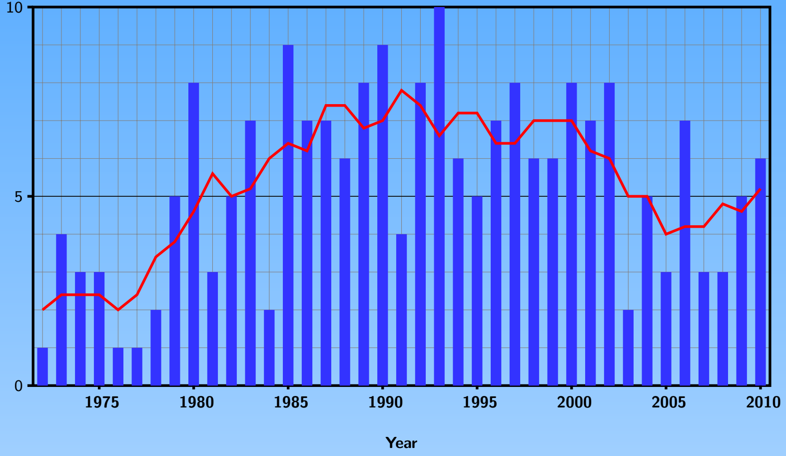
FREQUENT WORDS IN THE TWO BOOKS

Rank	AC	AofA	Rank	AC	AofA	Rank	AC	AofA
1	function	number	18	size	theorem	35	defined	size
2	number	trees	19	limit	permutations	36	words	recurrences
3	functions	exercise	20	theory	nodes	37	asymptotics	path
4	trees	analysis	21	figure	average	38	sequence	basic
5	theorem	tree	22	class	search	39	corresponding	proof
6	analysis	algorithms	23	tree	permutation	40	order	terms
7	analytic	chapter	24	probability	algorithm	41	labelled	general
8	set	binary	25	equation	first	42	section	because
9	combinatorial	generating	26	first	properties	43	instance	node
10	asymptotic	length	27	distribution	method	44	numbers	probability
11	generating	random	28	two	figure	45	saddle-point	gives
12	random	recurrence	29	general	consider	46	permutations	sum
13	case	function	30	chapter	numbers	47	large	combinatorial
14	singularity	table	31	simple	two	48	singularities	problem
15	form	functions	32	expansion	distribution	49	convergence	equation
16	coefficients	example	33	method	solution	50	proof	cycle
17	example	asymptotic	34	structures	values	51	series	cost

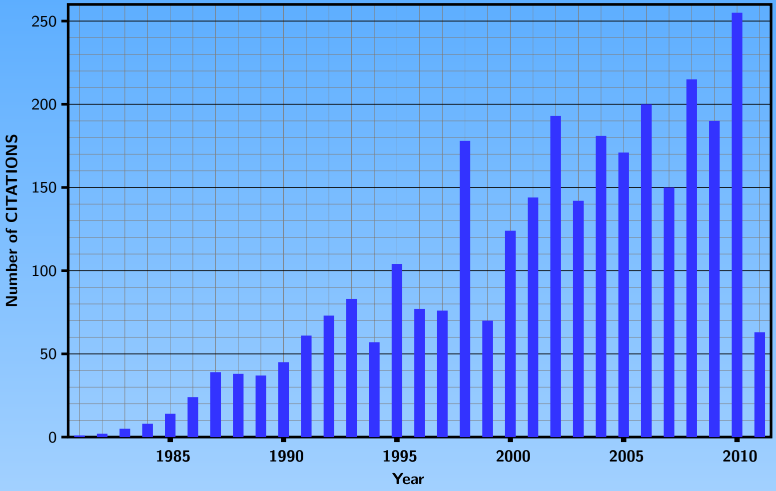
LE GRAND CHEF



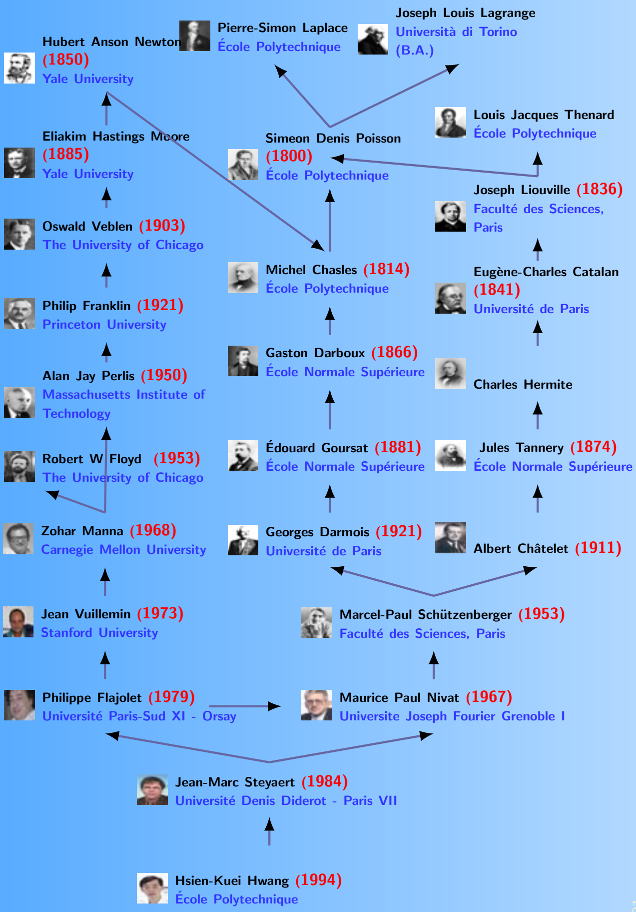
PUBLISHED ITEMS EACH YEAR (WEB OF SCIENCE)



CITATIONS EACH YEAR (WEB OF SCIENCE)



GENEALOGY



Hubert Anson Newton (1850)
Yale University
Pierre-Simon Laplace
École Polytechnique
Joseph Louis Lagrange
Università di Torino (B.A.)
Louis Jacques Thenard
École Polytechnique
Eliakim Hastings Moore (1885)
Yale University
Simeon Denis Poisson (1800)
École Polytechnique
Joseph Liouville (1836)
Faculté des Sciences, Paris
Oswald Veblen (1903)
The University of Chicago
Michel Chasles (1814)
École Polytechnique
Eugène-Charles Catalan (1841)
Université de Paris
Philip Franklin (1921)
Princeton University
Gaston Darboux (1866)
École Normale Supérieure
Charles Hermite
Alan Jay Perlis (1950)
Massachusetts Institute of Technology
Robert W Floyd (1953)
The University of Chicago
Édouard Goursat (1881)
École Normale Supérieure
Jules Tannery (1874)
École Normale Supérieure
Zohar Manna (1968)
Carnegie Mellon University
Georges Darmois (1921)
Université de Paris
Albert Châtelet (1911)
Jean Vuillemin (1973)
Stanford University
Marcel-Paul Schützenberger (1953)
Faculté des Sciences, Paris
Philippe Flajolet (1979)
Université Paris-Sud XI - Orsay
Maurice Paul Nivat (1967)
Universite Joseph Fourier Grenoble I
Jean-Marc Steyaert (1984)
Université Denis Diderot - Paris VII
Hsien-Kuei Hwang (1994)
École Polytechnique

COMPLEXITY THEORY AND FORMAL LANGUAGES

1972–1976: Almost all with Jean-Marc Steyaert

[1] **Complexité des problèmes de decision relatifs aux algorithmes de tri (ICALP'72)**

[2] **Decision problems for multihead finite automata (MFCS'73)**

[4] **A class of non recursive sorting algorithms (Journées Math. de la Compilation'73)**

[5] **Generalized immune sets (TR-IRIA'73; RAIRO'74 [7])**

[6] **On sets having only hard subsets (ICALP'74)**

[8] **Complexity classes of languages & operators (TR-IRIA'74)**

[10] **Linguistique formelle et linguistique historique (Info. et Philologie'75)**

[11] **Classes de complexité et reduction entre problemes (Codici'75; Liguori Pub. 1976 [12])**

LIFETIME INTERESTS AND ACHIEVEMENTS

jointly with Jean-Marc Steyaert

Une formalisation de la notion d'algorithme de tri non-récurrent, Thèse de 3e cycle, U. Paris VII, 1973, 289 pages.

Algorithms or Combinatorial Structures → **Languages** → **Complexity**

Languages

combinatorial, algebraic, analytic, probabilistic, continued fractions, integral transforms, orthogonal polynomials, singularity analysis, saddle-point method, contour integrals, . . .

Complexity

abstract and concrete, average-case analysis, analysis in distribution, asymptotic enumeration, . . .

AMBIGUITY AND TRANSCENDENCE

[46] Ambiguity & transcendence (ICALP'85)

[62] Prefixes(∞ words) & ambiguous context-free langs. (IPL'87)

[64] Analytic models & ambiguity of context-free langs. (TCS'87)

[83] Algebraically independent formal power series (ANT'90)

Idea

Context-free languages are inherently ambiguous if their counting GFs exhibit some characteristic form of transcendental behavior.

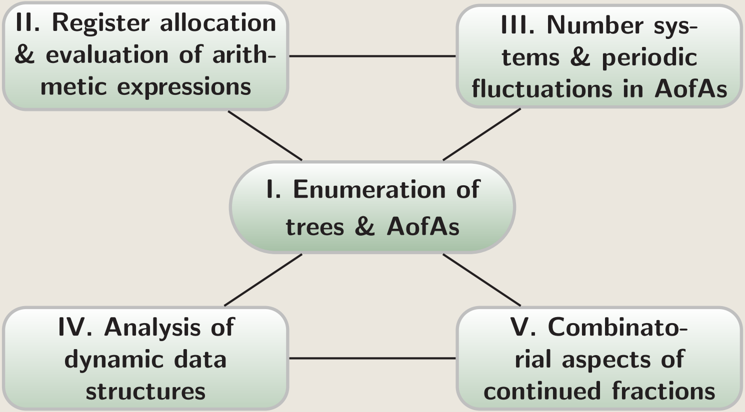
Criterion C. *If $l(z)$ has, in the vicinity of a singularity, an asymptotic equivalent that is not of the form*

$$\omega\left(1-\frac{z}{\alpha}\right)^{r}$$

with ω and α algebraic and r rational, then $l(z)$ is transcendental.

FROM (LANGUAGES, COMPLEXITY) TO (ANALYSIS OF ALGORITHMS, ANALYTIC COMBINATORICS)

Phd Thesis (Doctor of Science, Paris 11, 1979)
Analyse d'algorithmes de manipulation d'arbres et de fichiers
[14] [16] [29]



IV. ANALYSIS OF DYNAMIC DATA STRUCTURES

Histories(data types) ⇒ continued fractions & ortho. polys.

Tchebyshev	⟷	Stacks
Laguerre	⟷	Dictionaries
Chalier	⟷	Symbol Tables
Hermite	⟷	Priority Queues
Meixuer	⟷	Linear Lists

GF(integrated cost) ⇒ $\int$ GF(individual operation cost)

[18] **Sequences of operations for dynamic data structures (FOCS'79; STOC'79 [19]; ACCCC'80 [21]; Journées Algorithmiques'80 [24]; J. Algo'80 [25]; Questiió'81 [30])**

THEOREM 2,LL. *The exponential generating function $K(z)$ of integrated costs for linear lists is given by*

$$\hat{K}(z) = 2\int_0^{\mathrm{tg}^2 z/2} C(u) \frac{du}{u((1-u)^2 - 4u\cot^2 z)^{1/2}},$$

where

$$C(x) = \sum_{k\geq 0} (k+1)(CA_k + Cs_{k+1})x^{k+1}.$$

...the source of intriguing mathematical questions

Variance is also doable

[59] **Analysis of simple list structures (Info Sci'86)**

A problem of similar nature but different analysis

[56] **Evolution of two stacks (MFCS'86)**

V. COMBINATORIAL ASPECTS OF CONTINUED FRACTIONS

[22] **Combinatorial aspects of CFs (Ann DM'80, DM'80 (reprinted 2006) [23])**

Theorem 3A. *Let $P_{k,l,m}$ be the number of permutations having k minima (hence k+1 maxima), l double rises and m double falls. The generating function*

$$P(u,v,w,z)=\sum P_{k,l,m}u^k v^l w^m z^{2k+l+m+1}$$

has the expression:

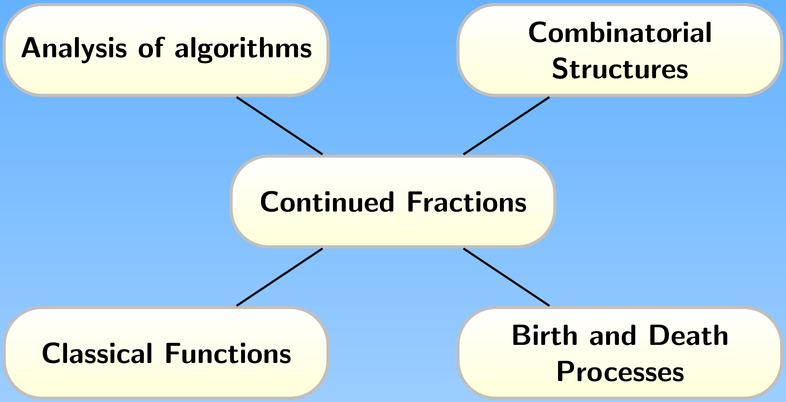
$$P(u,v,w,z)=\cfrac{1}{1-1(v+w)z-\cfrac{1\cdot 2uz^2}{1-2(v+w)z-\cfrac{2\cdot 3uz^2}{\cdots}}}$$

$$\sum_{n,k\geq 0} A_{n+1,k+1}u^k z^n=\cfrac{1}{1-1(1+u)z-\cfrac{1\cdot 2uz^2}{1-2(1+u)-\cfrac{2\cdot 3uz^2}{\cdots}}};$$

Another lifetime interest

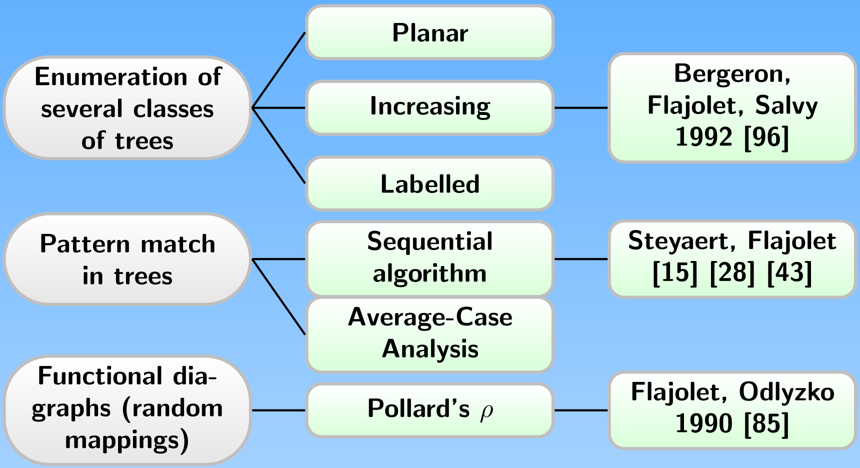
- [32] **Classical combinatorial #s: congruences and CFs (DM'82)**
- [77] **Elliptic fs., CFs & doubled perm. (Euro. JC'89)**
- [87] **Nonoverlapping partitions, CFs, Bessel functions and a divergent series (Euro. JC'90)**
- [144] **CF algorithms, functional operators, and structure constants (TCS'98)**
- [153] **Formal theory of birth-and-death processes, lattice path combinatorics, and CFs (AAP'00)**
- [186] **Fermat cubic, elliptic functions, CFs, and a combinatorial excursion (SLC'06)**
- [203] **Pseudo-factorials, elliptic functions, and CFs (Ramanujan J'10)**

CONTINUED FRACTIONS IN AOFA & AC



Classical analysis turned modern

I. ENUMERATION OF TREES & AOFAS



The road to perfection is long

I.2. PATTERN MATCHING IN RANDOM TREES & STRINGS

[15] **Analysis of tree matching (CAAP'78, ICALP'80 [28], Info&Ctrl'83 [43])**

Complexity calculus for tree algorithms

[31] **Complexity calculus for tree algorithms (FOCS'81, Math for CS'82 [35], Math Sys Th'87 [67])**

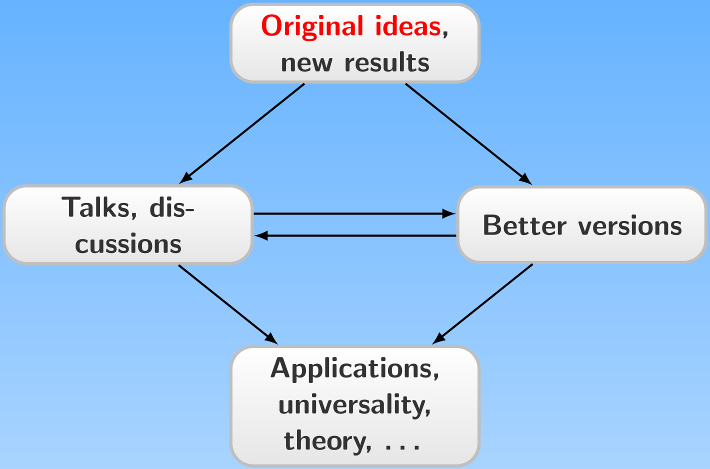
Other papers on pattern matching

[135] **Patterns in random binary search trees (RSA'97)**

[151] **Motif statistics (ESA'99, TCS'02 [174])**

[164] **Hidden pattern statistics (ICALP'01, JACM'06 [191])**

THE ROAD TO PERFECTION IS LONG AND WELL REHEARSED



III. NUMBER SYSTEMS & PERIODIC FLUCTUATIONS IN AOFAS

#(1's) in Gray code and analysis of odd-even merging

[27] **Gray codes and odd-even merge (SICOMP'80)**

$$\frac{1}{n}\sum_{k\geq 1}\frac{\binom{2n}{n-k}}{\binom{2n}{n}}\sum_{j<k} g(j) \sim \frac{1}{8}\log_4 n + \mathbf{Periodic}(\log_4 n)$$

Digital sums and divide-and-conquer recurrences

[105] **Exact asymptotics of divide-and-conquer recurrences (ICALP'93)**

[115] **Mellin transforms and asymptotics (Mergesort: Acta Info'94; Digital sums: TCS'95 [116])**

[199] **Multidimensional divide-and-conquer and weighted digital sums (ANALCO'09)**

DIVIDE-AND-CONQUER RECURRENCE

$$a_n = a_{\lfloor n/2 \rfloor} + a_{\lceil n/2 \rceil} + b_n$$

Most people's analysis

$$a_n \approx 2a_{n/2} + b_n \approx \sum_{j\geq 0} 2^j b_{n/2^j}.$$

Flajolet's view

$$a_n = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{n^{s+1}D(s)}{s(s+1)(1-2^{-s})}\mathbf{d}s, \qquad D(s) := \sum_{k\geq 1}\frac{\Delta\nabla b_n}{n^s}$$

Variance of top-down (or half-half) mergesort $\sim F(\log_2 n)n$

$$F(x) = \frac{1}{\log 2}\sum_{j\geq 1}\frac{2j(5j^2+10j+1)}{(j+1)(j+2)^2(j+3)^2}\log\frac{2j+1}{2j} + \frac{1}{\log 2}\sum_{k\neq 0}\frac{\Psi(\chi_k)e^{2k\pi i x}}{\chi_k(\chi_k+1)},$$

$$\Psi(s) := -\sum_{j\geq 1}\frac{2j(5j^2+10j+1)}{(j+1)(j+2)^2(j+3)^2}\left(\frac{1}{(2j)^s} - \frac{1}{(2j+1)^s}\right)$$

III.3. MELLIN TRANSFORMS

Importance of Mellin in describing periodic fluctuations

$$\sum_k \alpha_k \frac{\Delta^r \binom{2n}{n-k}}{\binom{2n}{n}} \sim n^{-r/2} B(x)$$

where $B(x) := \sum_k \alpha_k H_r(kx) e^{-k^2x^2}$**.**

$$B^*(s) = \frac{(-1)^r}{2} s(s-1)\cdots(s-r)\Gamma\left(\frac{s-r}{2}\right)\sum_{k\geq 1}\alpha_k k^{-s}$$

Two applications: $\alpha_k = v_2(k)^m$ **and** $\alpha_k = 2^{v_2(k)}$

[52] **Some uses of the Mellin integral transform in AofAs (NATO'85)**

[115] **Mellin transforms and asymptotics (Mergesort: Acta Info'94; Digital sums: TCS'94 [116]; Harmonic sums: TCS'95 [120]; Rice integrals: TCS'95 [124])**

II. AVERAGE NUMBER OF REGISTERS

[13] **Average number of registers required to evaluate arithmetic expressions (FOCS'77, TCS'79 [20])**

$$\mathrm{Reg}(T) := \max\{\mathrm{Reg}(T.\ell), \mathrm{Reg}(T.r)\} + 1_{\mathrm{Reg}(T.\ell)=\mathrm{Reg}(T.r)}$$

$$R_p(z) := \sum_n \#(\mathbf{Reg}(T_n) = p) z^n = \boxed{2i \sin\left(\phi \frac{e^{-2^p \phi}}{1 - e^{-2^{p+1}\phi}}\right)},$$

$$e^{-i\phi} = \frac{1-\sqrt{1-4z}}{2z} - 1$$

$$\mathbb{E}(\mathbf{Reg}(T_n)) = \sum_{j>0} v_2(j) \frac{\binom{2n}{n+j+1} - 2\binom{2n}{n+j} + \binom{2n}{n+j-1}}{\binom{2n}{n}\frac{1}{n+1}} \sim \log_4 n + \mathbf{Periodic}(\log_4 n)$$

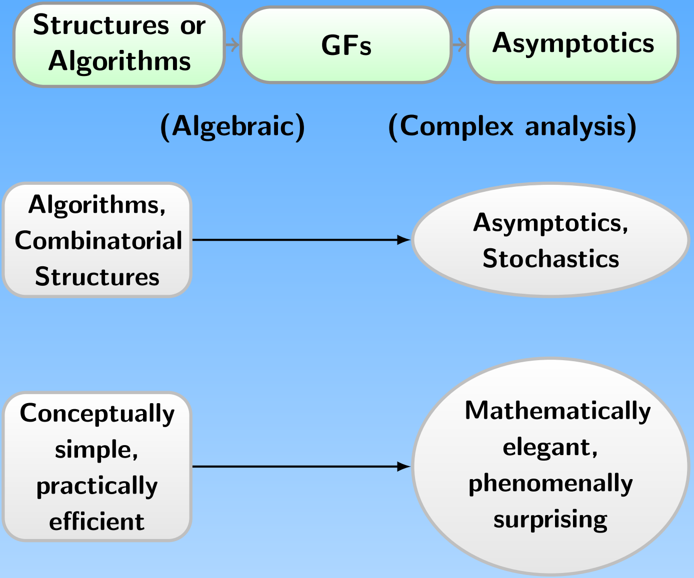
Unpublished

$$\frac{\#|T| = n; Reg(T) = p\}}{\frac{1}{n+1}\binom{2n}{n}} \sim \lambda\left(\frac{2p}{\sqrt{n}}\right), \text{ where } \lambda(x) = \sum_{k:odd} e^{-k^2x^2}(4k^2x^2 - 2)$$

[57] **Register allocation for unary-binary trees (SICOMP'86)**

BORN TO BE SUPER!

PATTERN OF ANALYSIS, STYLE



HEIGHT OF RANDOM TREES

$$\mathrm{H}(T) := \max\{\mathrm{H}(T.\ell), \mathrm{H}(T.r)\} + 1$$

[26] **Height of random binary trees (FOCS'80, JCSS'82 [33])**

[104] **LLT of heights of binary trees & other simple trees (CPC'93)**

[195] **Height of random binary unlabelled trees (DMTCS'08, RSA'11 [206])**

$$[z^n]Y_h(z) \sim ?, \; h \approx \sqrt{n}$$

$$Y_h(z) := \textstyle\sum_n \#(|T| = n, H(T) \le h) z^n$$

$$Y_{h+1}(z) = z\phi(Y_h(z)), \qquad Y_0(z) = 0.$$

$$\frac{y_n^{[h]} - y_n^{[h-1]}}{y_n} \sim \begin{cases} 2c\pi^{1/2} n^{-1/2} \beta^4 \sum\limits_{m \ge 1} (m\pi)^2 (2(m\pi\beta)^2 - 3) e^{-(m\pi\beta)^2} \\[2ex] 2c/(\beta\sqrt{n}) \sum\limits_{m \ge 1} m^2 (2(m/\beta)^2 - 3) e^{-(m/\beta)^2} \end{cases}$$

$$\beta := 2\sqrt{n}/(ch)$$

BINOMIAL SPLITTING PROCESSES

Tree structures

[34] A recursive partitioning process (WCMSM'82)

[53] Algebraic methods for trie statistics (Ann DM'85)

[36] A branching process (ICALP'82 [36])

[37] extendible hashing and trie searching (Acta Info'83)

[61] Digital search trees revisited (SICOMP'86)

[101] Digital trees & difference differential eqs. (RSA'92)

[140] The analysis of hybrid trie structures (SODA'98)

[161] Trie structures under dynamical sources (Algorithmica'01)

[187] The ubiquitous digital tree (STACS'06)

[208] Digital trees: from arithmetics to analysis (DMTCS'10)

BINOMIAL SPLITTING PROCESSES

Collision resolution algorithms

[49] A stack algorithm random multiple-access comm. (IEEE-IT'85)

[54] Collision resolution algorithms (IEEE-IT'85)

[55] A functional eq. in a multiaccess broadcast channel (AAP'86)

[68] Multiplicities of conflicts in multiple access channels (JACM'87)

[65] Analytic models for tree comm. protocols (FCCN'87, SMF'88 [71], PERFORMANCE'87 [72])

Sorting algorithms

[159] Bucket selection and sorting (Acta Info'00)

[202] #(symbol comparisons) in QuickSort and QuickSelect (ICALP'09)

BINOMIAL SPLITTING PROCESSES

Randomized counting schemes

[38] **Approximate counting** **(FOCS'83, BIT'85 [48])**

[40] **Probabilistic counting** **(CAAP'83, JCSS'85 [50])**

[42] **DSTs** **& exp variate generation (CAAP'83, J.Algo'86 [60])**

[84] **Adaptive sampling (Computing'90, Ency.'97 [134])**

[176] **LogLog counting** **of large cardinalities (ESA'03)**

[180] **Counting by** **coin tossings** **(ASIAN'04)**

[193] **Hyperloglog**: **a near-optimal cardinality estimation algo. (DMTCS'07)**

Estimate #(distinct elements in large files): Idea

Since 0^k1 **occurs with prob** 2^{-k-1}**, answer** 2^k **if see** 0^k1**,** k **largest.**
Many variations, fruitful developments and applications

The first use of saddle-point method in AofA

$$\pi_n^h = \frac{n!}{2^{hn}}[z^n]\left(\sum_{0\le j\le b}\frac{z^j}{j!}\right)^{2^h}$$

Theorem 1. *The probability that a b-trie (equiv. a directory in dynamic/extendible hashing with page capacity b) formed with n keys has height less than or equal to h satisfies*

$$\pi_n^h = \exp(-\beta 2^{bu(n)} 2^{-b\delta}) \cdot \left(1 + O\left(\frac{(\log n)^{b+1}}{n^{1/b}}\right)\right)$$

where $\beta = 1/(b+1)!$, $u(n)$ *is the fractional part of* $(1+1/b)\log_2 n$, $\delta = h - \lfloor (1+1/b)\log_2 n \rfloor$ *and h is in a "central" region around* $(1+1/b)\log_2 n$ *defined by*

$$\delta > -\log_2 \log n.$$

Furthermore, for fixed b, the $O(\cdot)$ *in the error term is uniform in h and n.*

TWO SNAPSHOTS

THEOREM 3.A. *The average value of parameter R_n satisfies:*

$$\bar{R}_n = \log_2(\varphi n) + P(\log_2 n) + o(1),$$

where constant $\varphi = 0.77351 \ldots$ *is given by*

$$\varphi = 2^{-1/2} e^{\gamma} \frac{2}{3} \prod_{p=1}^{\infty} \left[\frac{(4p+1)(4p+2)}{(4p)(4p+3)} \right]^{(-1)^{\nu(p)}}$$

larger than μ. *Then, each parameter* $\xi \in \{S, P, Q\}$ *has an expectation that satisfies asymptotically*

$$\mathbb{E}_{\mathcal{P}(x)}[\xi] = x A^{(\xi)}(\log x) + x\Phi^{(\xi)}(x) + O\left(x^{1-a}\right), \qquad x \to \infty, \tag{39}$$

for some $a > 0$, *where* $A^{(\xi)}$ *is polynomial given by* (38) *and* $\Phi^{(\xi)}$ *satisfies in each case the estimate*

$$\Phi^{(\xi)}(x) = O\left(\exp\left(-(\log x)^{1/(2\nu-1)}\right)\right). \tag{40}$$

Cost of prtial match queries

[41] **Analysis of partial match queries (FOCS'83, JACM'86 [58])**

[75] **Analysis of kdt-trees (WADS'89)**

[81] **Orthogonal range queries (WADS'89, Info Sci'89 [82])**

Quadtrees

[93] **Multidim. search in quadtrees (SODA'91, Algorithmica'93 [106])**

[103] **Page usage in a quadtree index (BIT'92)**

[122] **Hypergeometrics and the cost structure of quadtrees (RSA'95)**

[117] **Search costs in quadtrees and singularity perturbation asymptotics (DCG'94)**

PARTIAL MATCH QUERIES, *k-d* TREES

Linear systems, singularity analysis

THEOREM 1. *The average cost, measured by the number of internal nodes traversed, of a partial match query of specification pattern u in a k-d-tree constructed by random insertions from a file of size n satisfies*

$$c_{u,n} = \gamma_u n^{1-s/k+\theta(s/k)}[1 + o(1)],$$

where γ_u *is a strictly positive real constant*[2] *and the function* $\theta(x)$ *is defined as the unique positive real root in the interval* $[0; 1]$ *of the equation*

$$(\theta(x) + 3 - x)^x(\theta(x) + 2 - x)^{1-x} - 2 = 0,$$

so that, for $0 < x < 1$, $0 < \theta(x) < 0.07$.

The amount of work done in any partial match search with t keys specified in an ideal tree of n nodes is therefore $cn^{m/k} + d$ for some small constants c and d. This has been conjectured by Rivest [7] to be a lower bound for the average amount of work done in a partial match search; by construction we have shown this to be an upper bound not only for the average but for all partial match queries.

All of our analysis has been for the case of the perfectly balanced tree; the one in which we might expect to have the fastest searches. However, Rivest [7] has shown that the perfectly balanced trees have the *highest* average retrieval time. Therefore the results that we have shown are an expected upper bound on the retrieval time required by the algorithm.

Bentley (1975), $m = k - t$

QUADTREE RECURRENE

$$a_n = 2^d \sum_{0 \le j < n} \pi_{n,j} a_j + b_n$$

$$\pi_{n,j} = \binom{n-1}{j} \int_{[0,1]^d} (x_1 \cdots x_d)^j \, (1 - x_1 \cdots x_d)^{n-1-j} \, \mathrm{d}\mathbf{x}.$$

Exact solution

$$a_n = b_0 + n\Big((2^d - 1)b_0 + b_1\Big) + \sum_{2 \le k \le n} \binom{n}{k} (-1)^k [k]! \sum_{2 \le j \le k} \frac{b_j^\star - b_{j-1}^\star}{[j]!}$$

$$[n]! = \prod_{2 \le j \le n} \left(1 - \frac{2^d}{j^d}\right) \qquad b_n^\star = \sum_j \binom{n}{j} (-1)^j b_j$$

TWO SNAPSHOTS

Theorem 2. *The cost C_n of a random successful search in a standard quadtree of size $n-1$ has a generating function $\gamma_n(u)$ given by*

$$\gamma_n(u^2) \equiv \mathbf{E}\{u^{2C_n}\} = \frac{1}{n}\frac{u^2}{4u^2-1}\left[-1+\sum_{j=0}^{n}\binom{2u}{j}\binom{2u-1}{j}\binom{2u-1+n-j}{n-j}\right].$$

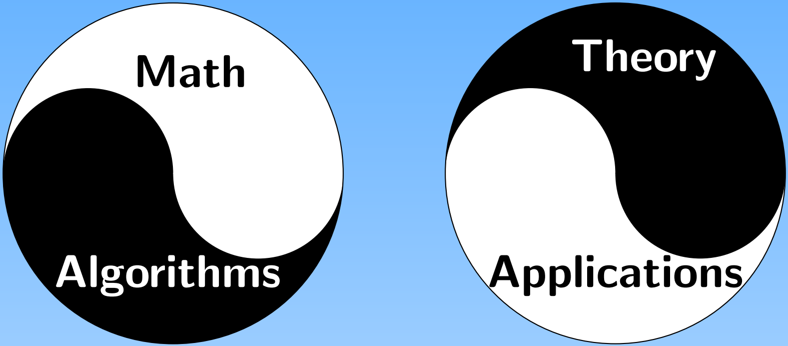
Theorem 3. *The expected internal path length of a quadtree of size n in dimension d is asymptotically*

$$f_n = \frac{2}{d}n\log n + \mu_d n + \mathcal{O}(\log n + n^{-1+2\cos(2\pi/d)}),$$

where μ_d depends only on the dimension:

$$\mu_d = \frac{1}{2d}(3d-3+4\gamma-4[\gamma]) = \frac{3d-3+4\gamma}{2d} - 2^{d+1}\sum_{m=3}^{\infty}\frac{1}{m(m^d-2^d)}.$$

SYNERGISTIC BALANCE



TREES, RANDOM MAPPINGS, SEQUENCES

[51] **Search trees and bubble memories (Wiener index) (RAIRO'85)**

[66] **Level number sequences for trees (DM'87)**

[89] **Common subexpression problem (tree compactification) (ICALP'90)**

[96] **Varieties of increasing trees (CAAP'92)**

[177] **And/or trees revisited (CPC'04)**

[198] **Isomorphism and symmetries in random phylogenetic trees (JAP'09)**

[74] **Deviations from uniformity in random strings (ZW'88, Algo&Comb'89 [76])**

[85] **Random mapping statistics (EUROCRYPT'89)**

LEVEL NUMBER SEQUENCES

Theorem 2. *The generating function of the quantities H_n is expressible as:*

$$H(q) = \frac{a(q)}{1-b(q)}, \tag{4}$$

where

$$a(q) = \sum_{j \geq 1} (-1)^{j+1} \frac{q^{2^{j+1}-2-j}}{(1-q)(1-q^3)(1-q^7)\cdots(1-q^{2^{j-1}-1})}, \tag{5}$$

$$b(q) = \sum_{j \geq 1} (-1)^{j+1} \frac{q^{2^{j+1}-2-j}}{(1-q)(1-q^3)(1-q^7)\cdots(1-q^{2^j-1})}. \tag{6}$$

Theorem 1. *The number of level number sequences H_n satisfies the asymptotic estimate:*

$$H_n \sim K \cdot \nu^n, \tag{2}$$

where $K = 0.254\,505\,523\,565\,319$ and $\nu = 1.794\,147\,187\,541\,685$ is the inverse of the smallest positive root ρ of the transcendental equation:

$$\sum_{j \geq 1} (-1)^{j+1} \frac{\rho^{2^{j+1}-2-j}}{(1-\rho)(1-\rho^3)(1-\rho^7)\cdots(1-\rho^{2^j-1})} = 1.$$

RANDOM GRAPHS, WALKS, TRIANGULATIONS, ETC.

Random graphs

[78] **First cycles in an evolving graph (DM'89)**

[154] **Density and robustness in random interconnection graphs (IFIP TCS'00; TCS'02 [172])**

Geometric objects

[141] **Maximum of a random walk (PEIS'98)**

[147] **Properties of random triangulations and trees (DCG'99)**

[149] **AC of non-crossing configurations (DM'99)**

[155] **AC of chord diagrams (FPSAC'00)**

[168] **AC of directed lattice paths (TCS'02)**

[204] **Asymptotics of 3-sided prudent polygons (J Phys A'10)**

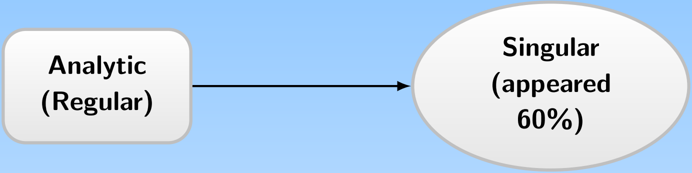
SINGULARITY ANALYSIS OF GENERATING FUNCTIONS

[86] Singularity analysis of generating functions (SIAM JADM'90)

[189] A hybrid of Darboux's method and singularity analysis in combinatorial asymptotics (EJC'06)

[148] Singularity analysis and asymptotics of Bernoulli sums (TCS'99)

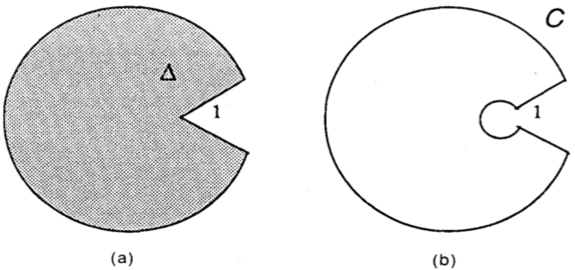
[182] Singularity analysis, Hadamard products, and tree recurrences (JCAM'05)



SINGULARITY ANALYSIS OF GENERATING FUNCTIONS

THEOREM 1. *Assume that, with the sole exception of the singularity $z = 1$, $f(z)$ is analytic in the domain $\Delta = \Delta(\phi, \eta)$, where $\eta > 0$ and $0 < \phi < (\pi/2)$. Assume further that as z tends to 1 in Δ,*

(2.6a) $$f(z) = O(|1-z|^{\alpha}),$$

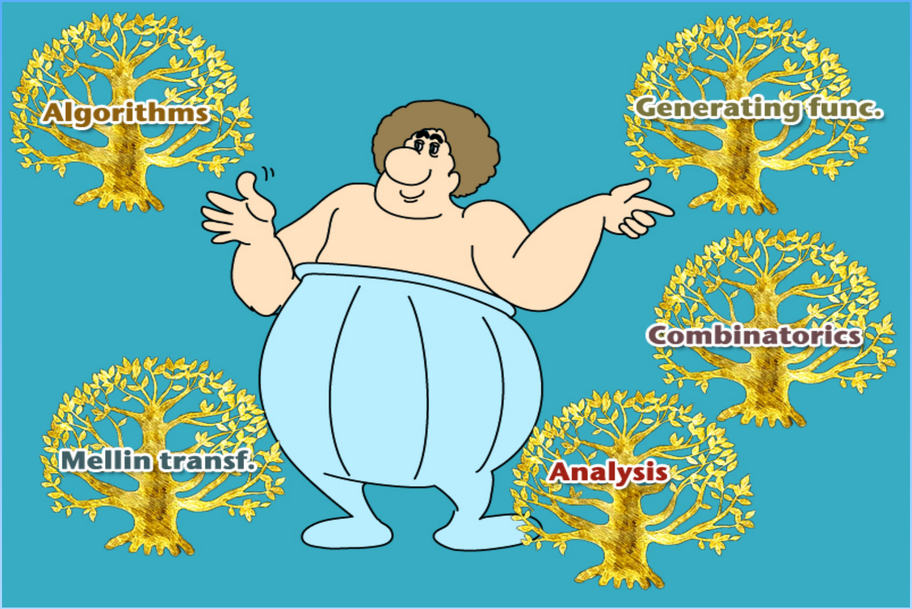


for some real number α. Then the nth Taylor coefficient of $f(z)$ satisfies

(2.6b) $$f_n = [z^n]f(z) = O(n^{-\alpha-1}).$$

Simple ideas, powerful analysis, far-reaching consequences

KING MIDAS


Algorithms
Generating func.
Combinatorics
Mellin transf.
Analysis

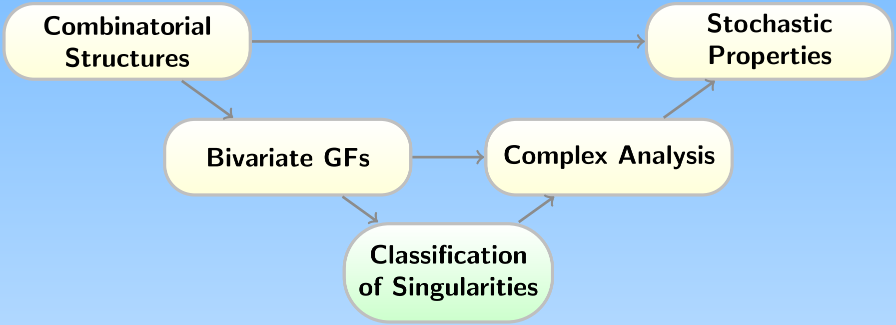
ANALYTIC SCHEMES FOR RANDOM COMBINATORIAL STRUCTURES

[45] **Asymptotic normality** of poly. iterations (Cambridge'84)

[88] $\mathcal{N}$ for #(components) in **comb. strucs.** (JCTA'90)

[112] **General combinatorial schemas (alg-log)** (DM'93)

[117] Quadtrees and singularity perturbation asymptotics (DCG'94)



ANALYTIC SCHEMES BY SINGULARITY ANALYSIS

Alg-log: $f(z,y) \approx \left(1 - \frac{z}{\rho(y)}\right)^{-\alpha} \implies \mathcal{N}(c_1 n, c_2 n)$

Exp-log: $f(z,y) \approx \left(1 - \frac{z}{\rho}\right)^{-\alpha(y)} \implies \mathcal{N}(c_1 \log n, c_2 \log n)$

Theorem 7. *Let $f_n(u)$ be a sequence of polynomials with positive coefficients satisfying the following conditions.*

C1. [Fixed regular singularity] *The generating function $F(u, z) = \sum_n f_n(u) z^n$ satisfies a linear differential equation of the form*

[117]

$$a_0(u, z)\frac{\partial^r F}{\partial z^r} + \frac{a_1(u, z)}{(1-z)}\frac{\partial^{r-1} F}{\partial z^{r-1}} + \cdots + \frac{a_r(u, z)}{(1-z)^r} F = 0,$$

where the $a_j(u, z)$ are polynomials and $a_0(u, z) \neq 0$ for $|z| \leq 1, |u| \leq 1$.

C2. [Nonconfluence] *The indicial equation*

$$a_0(1, 1)\alpha(\alpha - 1)\cdots(\alpha + r - 1) + \cdots + a_r(1, 1) = 0$$

has a root $\sigma > 0$ which is simple and such that all other roots $\alpha \neq \sigma$ satisfy $\Re(\alpha) < \sigma$.

C3. [Dominant growth] $f_n(1) \sim C \cdot b^{\sigma - 1}$ *for some $C > 0$.*

Then the coefficients of the polynomial $f_n(u)$ are asymptotically normal.

AUTOMATIC ANALYSIS OF ALGORITHMS

[63] **Mathematical tools for automatic program analysis (RR603, INRIA'87)**

[79] **Lambda Upsilon Omega (AAECC'88, RR'89 [80], TCS'91 [94])**

Operational vs conceptual

in a review: "mathematical methods are perhaps too often viewed here in a utilitarian perspective as "tools" rather than as theories having their own internal logic"

in an interview with MathMedia (to be published soon)

So notion of complexity is not only something utilitarian, . . . , but I think it is also conceptual, obviously certain property or deeply hidden in objects and still that's something we don't understand a lot philosophically and you need to do a lot in order to dig it out . . .

UNIVERSALITY OF AIRY PHENOMENA

[142] Analysis of **linear probing hashing** (Algorithmica'98)

[152] **Planar maps** and Airy phenomena (ICALP'00, RSA'01 [160])

[165] Analytic variations on the Airy distribution (Algorithmica'01)

[181] Airy phenomena and **analytic combinatorics** of connected graphs (EJC'04)

- Linear probing hashing
- Area under random walks
- Random maps
- # connected graphs
- Path lengths of random trees
- Inversions in trees
- 1st cycle in evolving graphs
- ...

GOOD AT CONNECTING DIFFERENT STRUCTURES

RANDOM GENERATION OF COMBINATORIAL STRUCTURES AND SIMULATIONS

- [113] **A calculus of random generation (ESA'93, TCS'94 [119])**
- [146] **Gen. funs. of generating trees (FPSAC'99, DM'02 [167])**
- [170] **Random sampling from Boltzmann principles (ICALP'02; CPC'04 [179])**
- [194] **Boltzmann sampling of unlabelled structures (ANALCO'07)**

Simulation of random variables

- [200] **On Buffon machines and numbers (SODA'11)**

URN MODELS

[183] Analytic urns (AP'05)

[188] Exactly solvable models of urn process theory (DMTCS'06)

[196] Analytic combinatorics of the Mabinogion urn (DMTCS'08)

Urn models → **PDE** → **Resolution of PDEs** → **Asymptotics**

THEOREM 1. *Consider the urn specified by matrix* $\begin{pmatrix} -a & a+s \\ b+s & -b \end{pmatrix}$*, with initial conditions* (a_0, b_0) *and* $t_0 := a_0 + b_0$*, assuming it to be tenable. The probability generating function at time n of the urn's composition is*

$$p_n(u) = \frac{\Gamma(n+1)\Gamma(t_0/s)}{s^n\Gamma(n+t_0/s)}[z^n]H(z,u),$$

where the bivariate generating function $H(z,u)$ *is given by*

$$H(z,u) = \delta(u)^{t_0}\psi\left(z\delta(u)^s + I(u)\right),$$

with

$$\delta(u) := (1-u^h)^{1/h}, \qquad I(u) := \int_0^u \frac{t^{a-1}}{\delta(t)^{a+b}}\,dt, \qquad h := a+b+s,$$

and the function ψ *is defined implicitly by*

$$\psi(I(u)) = \frac{u^{a_0}}{\delta(u)^{t_0}}.$$

NUMERICAL AND SYMBOLIC COMPUTATIONS

[107] **Une famille de polynômes (Gazette Math.'93)**

[109] **A finite $\sum$ of $\prod$s of binomial coefficients (SIAM Review'93)**

[121] **Ramanujan's Q-function (JCAM'95)**

$$\frac{21}{2} \leq \frac{4}{135\left(\sum_{0\leq j<n} \frac{n^j}{j!} - \frac{n!e^n}{2\cdot n!} - \frac{1}{3}\right)} - n \leq \frac{8}{45}$$

[123] **Computer algebra libs for combinatorial structures (JSC'95)**

[143] **Euler sums & $\int$ representations (Experimental Math.98)**

[197] **On differences of zeta values (JCAM'08)**

[185] **Fast computation of special resultants (JSC'06)**

[184] **Non-holonomic character of log, powers, and nth prime (EJC'05)**

[207] **Lindelöf representations and (non-)holonomic sequences (EJC'10)**

EULER SUMS AND INTEGRAL REPRESENTATIONS

(a) $\sum_{n\geq1}\frac{H_n}{n^2}=2\zeta(3),\qquad \sum_{n\geq1}\frac{H_n}{n^3}=\frac{5}{4}\zeta(4),\qquad \sum_{n\geq1}\frac{H_n}{n^4}=3\zeta(5)-\zeta(2)\zeta(3)$

(b) $\sum_{n\geq1}\frac{H_n^{(2)}}{n^4}=\zeta(3)^2-\frac{1}{3}\zeta(6)$

(c) $\sum_{n\geq1}\frac{H_n^{(2)}}{n^5}=5\zeta(2)\zeta(5)+2\zeta(3)\zeta(4)-10\zeta(7)$

(d) $\sum_{n\geq1}\frac{(H_n)^2}{n^5}=6\zeta(7)-\zeta(2)\zeta(5)-\frac{5}{2}\zeta(3)\zeta(4)$

(e) $\sum_{n\geq1}\frac{(H_n)^3}{n^4}=\frac{231}{16}\zeta(7)-\frac{51}{4}\zeta(3)\zeta(4)+2\zeta(2)\zeta(5)$

(f) $\sum_{n\geq1}\frac{(H_n)^4}{(n+1)^3}=\frac{185}{8}\zeta(7)-\frac{43}{2}\zeta(3)\zeta(4)+5\zeta(2)\zeta(5)$

(g) $\sum_{n\geq1}\frac{(H_n)^3}{n^5}-\frac{11}{4}\sum_{n\geq1}\frac{H_n^{(2)}}{n^6}=\frac{469}{32}\zeta(8)-16\zeta(3)\zeta(5)+\frac{3}{2}\zeta(2)\zeta(3)^2.$

(COMPUTATIONAL) NUMBER THEORY

[90] Gauss's reduction algorithm: An average case analysis (FOCS'90, ANTS'94 [114], CPC'97 [132])

[157] Continued fractions, comparison algorithms, and fine structure constants (CMSC'00)

Analysis of polynomial factorization algorithms

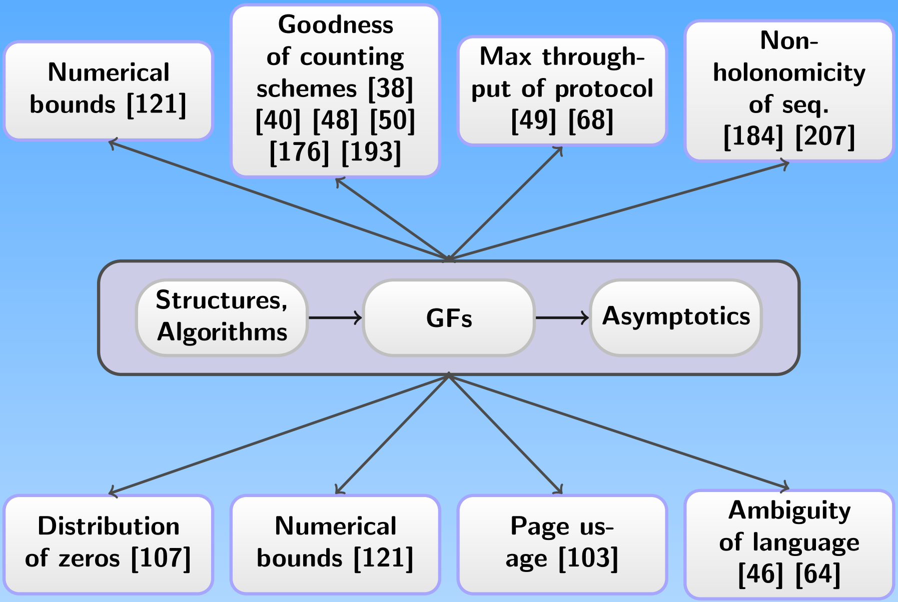
[145] Smooth polynomials over finite fields (ANTS'98)

[127] Random polynomials and polynomial factorization (ICALP'96, JALGO'01 [163])

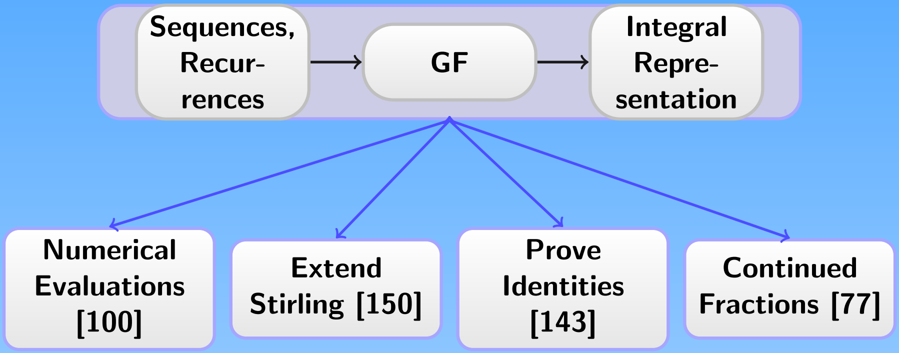
Mahler's partitions

[125] Asymptotique des récurrences mahleriennes (JTN Bordeaux'96)

IN ADDITION TO PROBABILISTIC PROPERTIES



IN ADDITION TO ASYMPTOTICS



2007 SODA lecture

The analytic combinatorial theory provides a powerful **conceptual framework** that could be summarized (optimistically) by the motto:

"If you can specify it, you can analyse it!".

COMBINATORICS AND GENERATING FUNCTIONS

- [92] Polya festoons (RR, INRIA 1991)
- [95] The cycle construction (SIDMA'91)
- [73] Probabilistic languages and random allocations (ICALP'88)
- [100] Birthday paradox, coupon collectors, caching algorithms, and self-organizing search (DAM'92)
- [150] Stirling numbers for complex argument (SIDMA'99)
- [205] Combinatorial models of creation-annihilation (TR arXiv:1010 0354, Arxiv'10)

Information theory

- [156] Analytic variations on the redundancy rate of renewal processes (IEEE Conf'2000; 2002 [173])
- [158] Data compression via binary decision diagrams (IEEE IT'00)

PROCEEDINGS EDITED

- [9] **Informatique et Philologie (IRIA, 1975)**
- [108] **Average Case Analysis of Algorithms (Dagstuhl Seminar Rep. 68, 1993)**
- [128] **Average Case Analysis of Algorithms (Dagstuhl Seminar Rep. 119, 1996)**
- [139] **Average-Case Analysis of Algorithms (RSA'97)**
- [169] **Mathematics and Computer Science II (Birkhäuser Verlag, Basel, 2002, 560 pages)**
- [178] **Mathematics and Computer Science III (Birkhäuser Verlag, 2004, 554 pages.)**

SURVEY, EXPOSITION, ESSAY, LECTURE NOTES

[17] **Deux problèmes d'analyse (Delange-Pisot-Poitou'79)**

[44] **Algorithmique (Encyclopedia Universalis 1984; Dictionnaire des mathématiques 1998)**

[47] **Elements of a general theory of comb. stru. (FCT'85)**

[69] **Math. methods in the AofAs and data structures (Trends in TCS'88)**

[70] **L'analyse d'algorithmes ou le risque calculé (Award lecture'86)**

[91] **Analysis of algorithms and data structures (Handbook TCS'90; 94p)**

[97] **Analytic analysis of algorithms (ICALP'92)**

[98] **La calculabilité et ses limites (La Science au Present'92)**

[99] **Introduction à l'analyse d'algorithmes (Singularité'92)**

SURVEY, EXPOSITION, ESSAY, LECTURE NOTES

[133] **Review of Micha Hofri's book "Analysis of Algorithms" (SIAM Review'97)**

[136] **SIGSAM Challenges: Symbolic asymptotics in practice (SIGSAM Bull.'97)**

[138] **Analysis of algorithms (RSA'97)**

[190] **Scientific works of Rainer Kemp (1949-2004) (TCS'06)**

[192] **Analytic combinatorics—a calculus of discrete structures (SODA'07)**

[162] $\mathbf{D} \cdot \mathbf{E} \cdot \mathbf{K} = (100)_8$ **(RSA'01)**

[171] **Singular combinatorics (ICM'02)**

[175] **Hachage, arbres, chemins, et graphes (Gaz. Math.'03)**

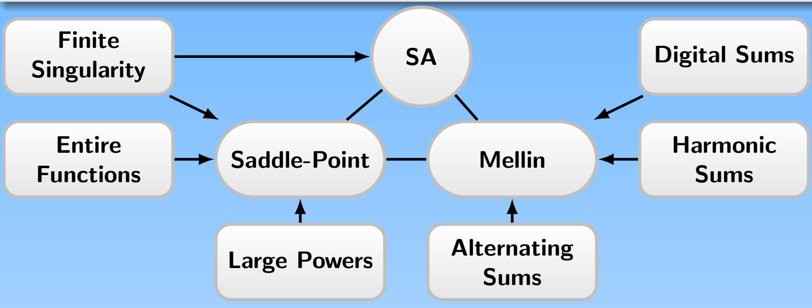
RESEARCH EASIER AFTER HIM

Theory (and Methodology): an analytic universe

Analytic tools → **Frameworks, Schemes** → **Systems**

founded a theory, the theory of Analytic Combinatorics

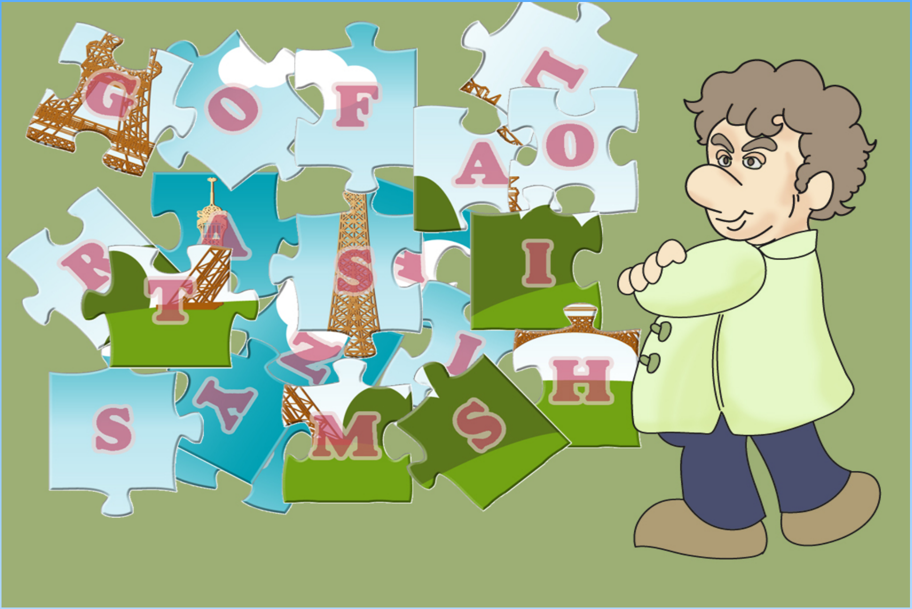
the avatar of Analytic Combinatorics



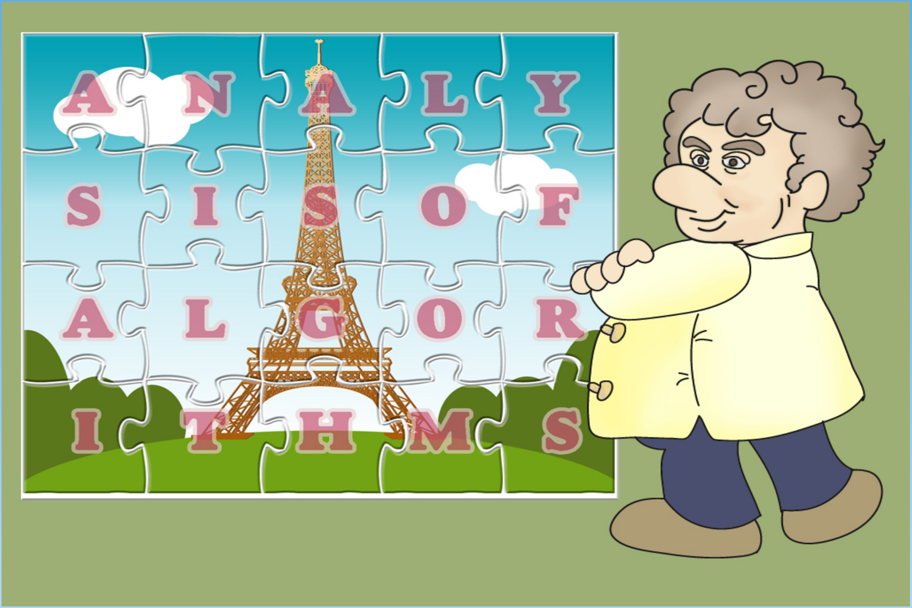
Style and taste: originality, depth, breadth

Key words: simple, efficient, elegant, surprising, general, systematic, precise, universal, conceptual, philosophical, . . .

ANALYSIS OF ALGORITHMS: OLD DAYS



ANALYSIS OF ALGORITHMS: NOW



We lost

an irreplaceable leader
a great scientist
a good friend

Luc Devroye

March 22, 2011. I do not care about the world today. Philippe Flajolet died this afternoon. My heart is in the gutter. Why is it that the nicest, smartest, most generous guys have to go first?

On my Facebook, I wrote: March 22, 2011—a day I will never forget. The world can go to hell. My friend, my brother, Philippe Flajolet just died. He was the guy I always wanted to be but never will be. Infinitely smart, generous, funny, radical, unconventional, creative, wonderful, wonderful, wonderful,. . . Thank you Philippe.

REPOSE EN PAIX

WE MISS YOU